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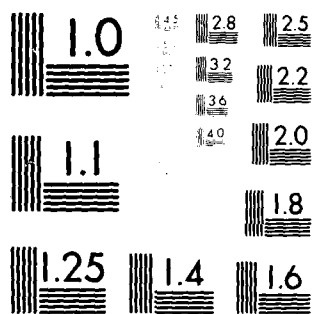
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USE OF SECOND ORDER STOCHASTIC
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Chelsea C. White
Andrew P. Sage

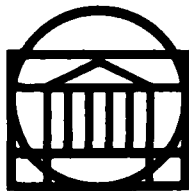
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guaranteed to contain the most preferred alternative on the basis of this assumption. The achievement of this objective presumably enhances decisionmaking since alternative selection is generally easier if made from a subset of the alternative set rather than from the entire alternative set.

The intent of this paper is to present an approach which achieves this objective and which has computational times amenable to interactive decision aiding. We make use of a fact, due to Fishburn and Vickson, which states that the feasibility of a certain collection of linear equalities and inequalities represents a necessary and sufficient condition for one alternative to be weakly preferred to another with respect to the second order stochastic dominance (SSD) relation. The approach presented here uses transitivity and upper and lower bounds on this relation in order to reduce the number of concomitant linear programs necessary for solution. The lower bound is provided by the first order stochastic dominance relation; the upper bound is given by a relation that is equivalent to the second order stochastic dominance relation when certain independence conditions hold. An example illustrates these results.

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USE OF SECOND ORDER STOCHASTIC DOMINANCE IN DECISION AIDING*

by

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ABSTRACT: In this paper, we examine a single-stage, multiobjective decisionmaking problem under uncertainty. The decisionmaker can select any one of a finite number of alternatives. After any alternative is chosen, one of a finite number of outcomes will result. The probabilistic relationship between each alternative and each outcome is presumed to be known. We assume that all that is known about the decisionmaker is that he or she is risk averse. Our objective is to determine the smallest subset of alternatives that is guaranteed to contain the most preferred alternative on the basis of this assumption. The achievement of this objective presumably enhances decisionmaking since alternative selection is generally easier if made from a subset of the alternative set rather than from the entire alternative set.

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INTRODUCTION

A well-known approach for multiobjective, single-stage decision aiding under uncertainty, c.f. (Keeney and Raiffa, 1976), has the following tasks associated with it:

- 1) Determine the utility function of the decisionmaker (DM).
- 2) Calculate the expected utility for each alternative.

(We assume throughout that the outcome probabilities as a function of alternative are given.)

- 3) Select the alternative having the largest expected utility.

A major difficulty with implementing this approach in practice is that utility function assessment is often a stressful task requiring a substantial amount of time and effort. Additionally, utility assessment can require cognitive perspectives not within the previous experience of the DM, which can produce results of lower quality than potentially achievable. A potentially useful tactic for reducing these difficulties is to investigate the implications of a less than complete description of the utility function, e.g. the various stochastic dominance procedures (Fishburn and Vickson, 1978).

Less than a complete description of the utility function, however, almost invariably produces less than a total ordering on the alternative set, and a weaker, partial ordering on the alternative set usually cannot identify the most preferred alter-

native. A partial order can, however, identify the nondominated set, a set of alternatives that is guaranteed under mild conditions to contain the most preferred alternative. The identification of the nondominated set is often quite adequate for decision aiding; e.g. see (White et. al, 1980) for a medical decisionmaking situation. We remark that the number of alternatives in the nondominated set is dependent on the amount of partial preference information known; see (White and Sage, 1980a, 1980b) for further discussion.

The apparent fact that the nondominated set is often a sufficiently informative aid for decisionmaking has motivated the development of decision aiding procedures that allow the mix of alternative order specificity and utility (or value) function identification time, effort and stress to be adaptively determined by the DM (White and Sage, 1980a, 1980b). Adaptive determination of this mix requires that the time necessary to determine the impact of additional (or initial) preference information on alternative order specificity be small due to the substantial constraints that are often placed on the available time of most DM's. One of the decision aiding procedures presented in (White and Sage, 1980b) is based on second order stochastic dominance (SSD) and hence on only the often quite behaviorally relevant assumption that the DM is risk averse. A straightforward implementation of this SSD-based decision aiding procedure requires the formulation and solution of $P(P-1)$ linear programs for a problem having P alternatives.

Our current computational experience indicates that such an implementation almost invariably requires computation times that are unacceptably large, even for small problems, for interactive decision aiding. The intent of this paper is to propose a set of procedures that may significantly reduce the computational time and effort necessary to order the alternative set using SSD, thus enhancing the potential of the SSD-based approach as an interactive decision aiding procedure.

This paper is organized as follows. The problem is formulated and preliminary results and definitions are given in Section 2. In Section 3, we state and investigate conditions that may often reduce the time required by the approach taken in (White and Sage, 1980b) to order the alternative set using SSD. Conclusions are presented in the final section.

PROBLEM FORMULATION AND PRELIMINARIES

We assume that the DM can select for implementation any one of P predetermined alternatives from the alternative set $\Pi = \{\pi^1, \dots, \pi^P\}$. After an alternative is implemented, any one of M possible outcomes will occur. There are N objectives under consideration. Let v_n^m be the predetermined value score of the m^{th} outcome with respect to the n^{th} objective. The real number v_n^m is isotone (monotonically nondecreasing) in preference with respect to the n^{th} objective; that is, outcome m' is (weakly) preferred to outcome m with respect to objective n if and only if $v_n^{m'} \geq v_n^m$. Let $V = \{v^m, m = 1, \dots, M\}$, the set of all value score vectors, where $v^m = \{v_1^m, \dots, v_N^m\}$. The probability that outcome m will result if alternative π^P is selected is $\pi^P(v^m)$.

We assume that there exists a (presumably unassessed) utility function $u: V \rightarrow \mathbb{R}$ which reflects the DM's preferences in that outcome m' is (weakly) preferred to outcome m with all objectives under consideration if and only if $u(v^{m'}) \geq u(v^m)$. Alternatives are compared on the basis of expected utility: alternative π' is (weakly) preferred to alternative π if and only if $E(u, \pi') \geq E(u, \pi)$, where

$$E(u, \pi) = \sum_{v \in V} u(v) \pi(v).$$

We make the following two assumptions about the DM:

- 1) Assume that the DM prefers outcome m' to outcome m when only objective n is considered. Let this statement be true for all $n = 1, \dots, N$. Then, the DM prefers outcome m' to outcome m when considering all objectives simultaneously (consistency).
- 2) The DM prefers the expected consequence of a lottery (the concept of a lottery is discussed at length in (Keeney and Raiffa, 1976)) to that lottery (risk aversion).

These two assumptions imply that the DM's utility function is isotone and concave, respectively, and thus is a member of the set $U_2 = \{u: u \text{ is isotone and concave}\}$. The objective is to provide the DM with a set of alternatives which is guaranteed to contain the most preferred alternative under the assumption that all that is known about the DM's utility function is that it is a member of U_2 .

We say that alternative π' is (weakly) preferred to alternative π with respect to SSD, i.e. $\pi' R_2 \pi$, if and only if $E(u, \pi') \geq E(u, \pi)$ for all $u \in U_2$. Thus, if the DM is consistent and risk averse, $\pi' R_2 \pi$ implies that π' can be expected to be at least as good an alternative choice as π . It is well-known, c.f.

(White and Sage, 1980a), that the most preferred alternative is a member of the nondominated set $\{\pi \in \Pi: \text{there does not exist a } \pi' \in \Pi \text{ such that } \pi' R_2 \pi \text{ and not } \pi R_2 \pi'\}$. Determining the nondominated set of Π with respect to R_2 , or more generally

determining the set $P_2 \subseteq \Pi \times \Pi$, where $(\pi', \pi) \in P_2$ if and only if $\pi' R_2 \pi$, helps to restrict the search for the most preferred alternative, thus presumably aiding decisionmaking. (Note that π is nondominated if there is no $\pi' \in \Pi$ such that $(\pi', \pi) \in P_2$ and $(\pi, \pi') \notin P_2$; thus, knowledge of P_2 can be used to determine the nondominated set of Π with respect to R_2 .)

The following necessary and sufficient conditions, a slightly generalized version of a result presented in (Fishburn and Vickson, 1978), suggest an approach for determining P_2 .

THEOREM 1: $\pi' R_2 \pi$ if and only if there exists a feasible solution to the set of linear equalities and inequalities:

- (i) $d_{ij} \geq 0$ for all $i, j = 1, \dots, M$
- (ii) $\sum_{j=1}^M d_{ij} = 1$ for all $i = 1, \dots, M$ such that $\pi'(v^i) \neq 0$
- (iii) $\pi(v^j) = \sum_{i=1}^M \pi'(v^i) d_{ij}$ for all $j = 1, \dots, M$
- (iv) $\sum_{i=1}^M d_{ij} v_n^j \leq v_n^i$ for all $i = 1, \dots, M$ such that $\pi'(v^i) \neq 0$ and all $n = 1, \dots, N$.

A probabilistic interpretation of $\{d_{ij}\}$ is given in (Fishburn and Vickson, 1978).

A brute-force application of Theorem 1 for determining P_2 requires the formulation and solution of $P(P-1)$ linear programs,

each having up to $M(M + N + 2)$ decision variables, $2M$ of which are artificial variables, and up to $M(N + 2)$ side constraints. Our present level of experience with the process of formulating and solving these linear programs indicates that they may often require computer time that is unacceptably large for interactive decision aiding, even with relatively small problems and a relatively fast computer. For example, the $P=6$, $M=5$, and $N=4$ problem considered in Example 3 (White and Sage, 1980b) required roughly 1.5 minutes of CPU time on the CDC-6400 at the Computing Center of the University of Virginia. Since our computations were done interactively in a time sharing mode; turn around time can be expected, and was experienced, to be between 5 and 15 minutes, depending on the system load. This length of time seems excessive for interactive decision aiding. In the next section, we present techniques that may often reduce, sometimes substantially, this computational burden.

MAIN RESULTS

In the previous section, a procedure for determining P_2 was suggested that involved a straightforward application of Theorem 1. In this section, we present several results that may often reduce, sometimes substantially, the computational times associated with this procedure. These results are

presented and proved following three preliminary definitions.

1. The alternative π' is said to be (weakly) preferred to π with respect to first order stochastic dominance (FSD), i.e. $\pi' R_1 \pi$, if and only if $E(u, \pi') \geq E(u, \pi)$ for all $u \in U_1 = \{u: u \text{ is isotone}\}$.

2. The alternative π' is said to be (weakly) preferred to π with respect to strong-SSD (\bar{R}_2), i.e. $\pi' \bar{R}_2 \pi$, if and only if, for each $n = 1, \dots, N$, there exists a feasible solution to the set of linear equalities and inequalities (i), (ii), (iii), and

$$(iv)' \quad \sum_{i=1}^M d_{ij} v_n^j \leq v_n^i \quad \text{for all } i=1, \dots, M \text{ such that } \pi'(v^i) \neq 0.$$

3. An arbitrary relation R_b on Π is said to be stronger than (more precisely, at least as strong as) an arbitrary relation R_a on Π , i.e. $R_a \subseteq R_b$, if and only if $\pi' R_a \pi$ implies $\pi' R_b \pi$, for any pair $\pi', \pi \in \Pi$.

We remark that $\pi' \bar{R}_2 \pi$ is equivalent to N separate checks for univariate SSD, for which there exists a computationally simple procedure, c.f. Section 2.14 in (Fishburn and Vickson, 1978). We also note that under certain independence conditions (presented, for example, in Theorem 2.11 of (Fishburn and Vickson, 1978)), $R_2 = \bar{R}_2$, i.e. $R_2 \subseteq \bar{R}_2$ and $\bar{R}_2 \subseteq R_2$. We now present our

main result, the proof of which is in the Appendix.

THEOREM 2: R_1 , R_2 , and \bar{R}_2 are transitive, and $R_1 \subseteq R_2 \subseteq \bar{R}_2$.

The impact of this result is due to the fact that it can be used to reduce, often drastically, the number of linear programs that require solution in order to construct the set P_2 . Transitivity implies that it is not necessary to check whether or not $\pi'' R \pi$, if $\pi'' R \pi'$ and $\pi' R \pi$, for arbitrary transitive relation R . $R_1 \subseteq R_2 \subseteq \bar{R}_2$ implies that once P_1 and \bar{P}_2 are known (where we define P_1 and \bar{P}_2 similarly to P_2), the only pairs that require the linear program check in order to complete determination of P_2 are those pairs in \bar{P}_2 which are not in P_1 and which have not already been added to P_2 by the above transitivity argument. Theorem 2, therefore, suggests the following four step procedure for determining P_2 :

- (1) Determine P_1 . (Relatively computationally simple procedures for determining P_1 are presented in (White and Sage, 1980b).)
- (2) Determine \bar{P}_2 . (Relatively computationally simple procedures for determining \bar{P}_2 are suggested in (Fishburn and Vickson, 1978).)
- (3) Evaluate all pairs in P_1 which are not in \bar{P}_2 , using Theorem 1.
- (4) Construct P_2 by adding the appropriate pairs from Step 3 to P_1 .

The above procedure prompts three comments. First, the transitivity of all relations can be useful in reducing the number of linear program formulations and solutions, a fact not explicitly mentioned above. Second, it is inconsequential that Step 1 is performed prior to Step 2. Whichever of the first two steps is performed first can, however, impact on the time required to perform the second step. This impact is due to the facts that if $(\pi', \pi) \in P_1$, then $(\pi', \pi) \in \bar{P}_2$ and if $(\pi', \pi) \notin \bar{P}_2$, then $(\pi', \pi) \notin P_1$, since $P_1 \subseteq \bar{P}_2$. Third, in checking for strong-SSD, it is necessary to examine only down to the first objective that fails to satisfy the univariate SSD criterion (if one exists).

We now present an example illustrating the above procedure.

EXAMPLE: Consider Example 3 in (White and Sage, 1980b). In that example problem, there were six available alternatives, five possible outcomes, and four objectives under consideration prior to the objective aggregation procedure, i.e. $P=6$, $M=5$, and $N=4$. The Table presents the assumed data. Results in (White and Sage, 1980b) indicate that $P_1 = \{(4,3)\}$. Calculations based on procedures suggested in Section 2.14 of (Fishburn and Vickson, 1978) show that $\bar{P}_2 = \{(1,3), (2,3), (4,3)\}$. Thus, it is only necessary to check the pairs (1, 3) and (2, 3) in order to determine P_2 . Solution of the two associated linear programs

	Outcome Number				
	m=1	m=2	m=3	m=4	m=5
Objective Number					
n=1	10	5	5	0	5
n=2	10	0	0	0	0
n=3	3	3	10	0	3
n=4	5	5	5	0	10

(a) Value Scores for Each Outcome and Objective

	Outcome Number				
	m=1	m=2	m=3	m=4	m=5
p=1	0.6	0.1	0.2	0.1	0.0
p=2	0.7	0.0	0.1	0.2	0.0
p=3	0.3	0.1	0.0	0.4	0.2
p=4	0.3	0.0	0.1	0.1	0.5
p=5	0.1	0.1	0.0	0.1	0.7
p=6	0.0	0.1	0.1	0.0	0.8

(b) Outcome Probabilities for Each Alternative

Table: Data for the Example

indicates that $(1, 3) \in P_2$ and $(2, 3) \notin P_2$, and hence $P_2 = P_1 \cup (1, 3) = \{(1, 3), (4, 3)\}$. Note that $P_2 \neq \bar{P}_2$. This result is in agreement with a result found in (White and Sage, 1978), which was determined from the formulation and solution of $P(P-1) = 6(6-1) = 30$ linear programs.

The first three objectives were linearly aggregated in Example 3 (White and Sage, 1980b) with weights 0.1, 0.1, and 0.8, respectively. As a result, $P_1 = \{(1,2), (4,3), (6,5)\}$. Calculations show that $\bar{P}_2 = \{(1,2), (1,3), (2,3), (4,3), (6,5), (6,3), (5,3)\}$ (note that $(1,3)$ and $(6,3)$ are members of \bar{P}_2 by transitivity). Thus, the only pairs that require examination by the procedure suggested in Theorem 1 are: $(1,3), (2,3), (6,3), (5,3)$. We observe that if $(2,3) \in P_2$ and $(5,3) \in P_2$, then it is not necessary to check if $(1,3) \in P_2$ and $(6,3) \in P_2$, respectively, because of the transitivity of P_2 . Solution of the associated linear programs show that $(1,3)$ and $(6,3)$ are members of P_2 . We have therefore determined that $P_2 = \bar{P}_2$ by formulating and solving only two linear programs. This result is in agreement with a result found in (White and Sage, 1980b), which again was determined from the formulation and solution of 30 linear programs.

CONCLUSIONS

This paper has investigated procedures for making SSD a viable concept for interactive decision aiding. Our primary contribution toward achieving this objective has been the identification of a partial order that acts as an upper bound on the SSD partial order. Our present level of experience indicates that this upper bound, the FSD partial order lower bound, the transitivity of all three partial orders, and necessary and sufficient conditions due to (Fishburn and Vickson, 1978) can often be used to obtain a significant reduction in the computational demands associated with a straightforward application of Theorem 1 in determining P_2 .

The four step procedure for determining P_2 proposed here, however, may not always reduce computational time. Although straightforward application of Theorem 1 requires formulating and computing at least as many linear programs as required by the four step procedure; it does not require the determination of P_1 , \bar{P}_2 , or transitivity checks. If $P_1 = \phi$ and $\bar{P}_2 = \Pi \times \Pi$, it is clear that an application of Theorem 1 that allows for transitivity checks will be computationally quicker and therefore superior to the four step procedure. We have found, however, that the number of pairs in \bar{P}_2 which are not in P_2 have usually been a small fraction of $P(P-1)$ and that the time

necessary to calculate P_1 and \bar{P}_2 has typically been significantly smaller than the time necessary to formulate and calculate the additional linear programs. Future computational experience is expected to further indicate the merits of both approaches for calculating P_2 .

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Appendix: Proof of Theorem 2.

It is shown in (Fishburn and Vickson, 1978) that R_1 and R_2 are transitive and that $R_1 \subseteq R_2$. A simple argument, based on the fact the univariate SSD relation is transitive, proves that \bar{R}_2 is transitive. In order to prove $R_2 \subseteq \bar{R}_2$, define \mathcal{D}_0 as the set of all $\{d_{ij}\}$ such that (i), (ii), and (iii) hold and \mathcal{D}_n as the set of all $\{d_{ij}\}$ such that (iv)' holds. Note that:

(a) $\pi^* R_2 \pi$ if and only if

$$\mathcal{D}_0 \cup \left[\bigcap_{n=1}^N \mathcal{D}_n \right] \neq \phi$$

(b) $\pi^* \bar{R}_2 \pi$ if and only if

$$\mathcal{D}_0 \cap \mathcal{D}_n \neq \phi \text{ for all } n = 1, \dots, N.$$

Use of the fact that

$$\mathcal{D}_0 \cap \left[\bigcap_{n=1}^N \mathcal{D}_n \right] = \bigcap_{n=1}^N \left[\mathcal{D}_0 \cap \mathcal{D}_n \right]$$

easily implies $R_2 \subseteq \bar{R}_2$.

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School of Engineering and Applied Science

The University of Virginia's School of Engineering and Applied Science has an undergraduate enrollment of approximately 1,400 students with a graduate enrollment of approximately 600. There are 125 faculty members, a majority of whom conduct research in addition to teaching.

Research is an integral part of the educational program and interests parallel academic specialties. These range from the classical engineering departments of Chemical, Civil, Electrical, and Mechanical and Aerospace to departments of Biomedical Engineering, Engineering Science and Systems, Materials Science, Nuclear Engineering and Engineering Physics, and Applied Mathematics and Computer Science. In addition to these departments, there are interdepartmental groups in the areas of Automatic Controls and Applied Mechanics. All departments offer the doctorate; the Biomedical and Materials Science Departments grant only graduate degrees.

The School of Engineering and Applied Science is an integral part of the University (approximately 1,530 full-time faculty with a total enrollment of about 16,000 full-time students), which also has professional schools of Architecture, Law, Medicine, Commerce, Business Administration, and Education. In addition, the College of Arts and Sciences houses departments of Mathematics, Physics, Chemistry and others relevant to the engineering research program. This University community provides opportunities for interdisciplinary work in pursuit of the basic goals of education, research, and public service.

